ISI – Bangalore Center – B Math - Physics II – Back paper Exam
Date: 27 December 2016. Duration of Exam: 3 hours
Total marks: 50

ANSWER ALL QUESTIONS

Q1. [Total Marks: 3+3+2+2=10]

a.) Using only the first law of thermodynamics show that for any gas that can be described by a PVT system

$$C_p - C_v = \left[\left(\frac{\partial U}{\partial V}\right)_p + P\right]\left(\frac{\partial V}{\partial T}\right)_p$$ where U is the internal energy.

b.) Using the second law of thermodynamics, and Maxwell relations show that the above can be written as

$$C_p - C_v = T\left(\frac{\partial P}{\partial T}\right)_P\left(\frac{\partial V}{\partial T}\right)_p$$

c.) Show that $\left(\frac{\partial P}{\partial T}\right)_p = -\left(\frac{\partial P}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p$.

d.) Prove that $C_p - C_v \geq 0$.

Q2. [Total Mark:5+3=8]

a.) State the Kelvin-Planck and the Clausius statements for the 2nd law of thermodynamics and show they are equivalent.

b.) Show if two adiabatic curves of a P, V, T system intersect, it will violate the second law of thermodynamics.

Q3. [Total Marks:6+4=10]

a.)A balloon filled with an ideal gas is at temperature T_i and is kept at constant pressure. It is allowed to interact with a reservoir at temp $T < T_i$ until the temperature of the gas in the balloon becomes same as the temperature of the reservoir. In the process, the volume of the gas changes from V_i to V. Show that the change in entropy of the gas is given by

$$S(V,T) - S(V_i,T_i) = \int_{T_i}^{T} C_v \frac{dT}{T} + +R\ln\frac{V}{V_i}$$

b.) Explain in words, without detailed calculation, why the change of entropy calculated above is negative whereas as one would expect that the change of entropy associated with a spontaneous change such as the above is positive.

Q3. [Total Marks:2+5+1+2=10]

a.) Define the partition function of a system in thermal equilibrium at temperature T.

c.) Calculate the partition function of a one dimensional system of N distinguishable particles, where each particle can have energy $(n+\frac{1}{2})\hbar\omega$ where n is an integer . Show that the average energy is given by $N\frac{\hbar\omega}{2}\left(1+\frac{2e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}}\right), \beta=\frac{1}{kT}$.

d.) What are the high temperature and low temperature limits of the average energy?

e.) How will the result for average energy change if the system is considered as a three dimensional system with energy given by $(m+n+p+\frac{3}{2})\hbar\omega$ where m, n, p are all integers. Hint: You need not do a detailed calculation again!

Q5.[Total Marks:2+4+6=12]

a.) In a two slit interference with identical coherent sources with wavelength λ, let Δ be the phase difference between the rays that generate an interference pattern at one point on the screen. Let Δ' be the corresponding phase difference at another point in the interference pattern. What is $\Delta'-\Delta$ if these two points belong to two successive maxima? What is $\Delta'-\Delta$ if these two points belong to a maximum and next to next minimum?

b.) In the above experiment using 5000Å light it is observed that the central fringe moves to a spot previously occupied by the 4^{th} order bright fringe when a transparent material with refractive index 1.2 is placed in front of one of the slits. What is the thickness of the material?

c.) Consider instead of two slits, three slits where the slits are apart by *d* and *3d/2* as shown in the Figure 1. (The figure is not to scale, the screen distance is much larger than *d*). Asumme the these are coherent sources with amplitude *A*.

Show that the intensity on the screen is given by

$$I= A^2\left(3+2\left[\cos\delta+\cos\frac{3\delta}{2}+\cos\frac{5\delta}{2}\right]\right) \text{ where } \delta=\frac{2\pi d\sin\theta}{\lambda}.$$

What is the value of θ where first principal maximum occurs?

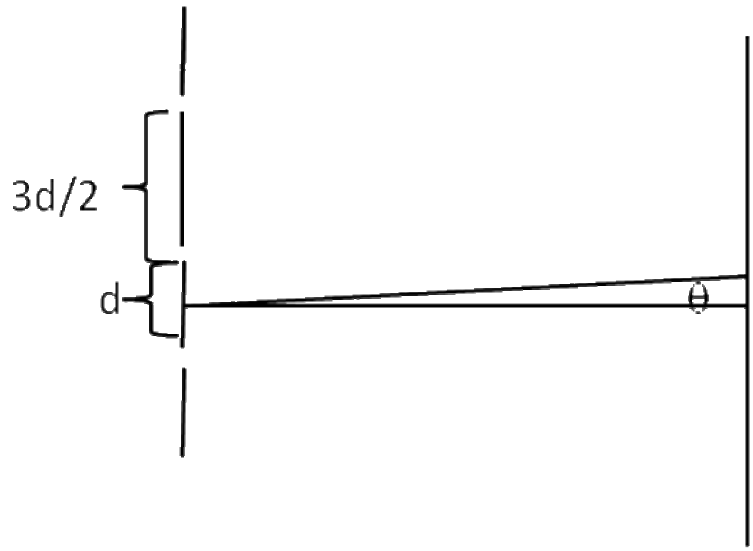

Figure 1. (Not drawn to scale. Distance to the screen is large compared to *d*.)

Maxwell Relations that you may use or not use:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$